TAIWAN ECONOMIC
FORECAST AND POLICY
43:2 (2013), 23–49

Estimating Potential Output for Taiwan with Seasonally Unadjusted Data

Jin-Lung Lin*
Department of Finance
National Dong Hwa University

Shin-Hui Chen
Department of Economics
National Dong Hwa University
and Center for Economic Forecasting
Chung-Hua Institution for Economic Research

Keywords: Potential GDP, Output gap, NAIRU, Seasonal unit root, Okun's law, Phillips curve
JEL Classification: E23, E24, E62

---

* Correspondence: Jin-Lung Lin, Department of Finance, National Dong Hwa University, Hualien 974, Taiwan. Tel: (03) 863-3146; Fax: (03) 863-3130; E-mail: jlin@mail.ndhu.edu.tw.
I would like to thank National Science Council, Taiwan, for providing financial support under the project NSC-95-2415-H-001-021. We are grateful to two anonymous referees for helpful comments. Any remaining errors are solely our responsibility.

ABSTRACT

Measuring potential output and the output gap have long been an important task for conducting monetary and fiscal policies. There exist several methods for this purpose and a partial list includes the univariate detrending method, the multivariate filtering approach, and the structural VAR system approach. One common feature of all these methods is assuming the existence of a unit root for the unobserved potential GDP. While this assumption is appropriate for the cases of US and most European countries where macroeconomic data are seasonally adjusted, it does not fit the Taiwanese economy. Almost all of Taiwan's macroeconomic data are seasonally unadjusted, and the seasonal unit root as well as richer dynamics have to be embedded in the model. In this paper, we analyze the impact of seasonality on various potential output measures. To check robustness and investigate how sensitive the results are to further changes in the specification of the NAIRU and the unemployment gap, distinct classes of NAIRU and unemployment gap concept are implemented. Empirical analysis confirms the importance of seasonal behavior. Switching from a regular unit root to a seasonal unit root improves the efficiency of measuring potential GDP and output gap for Taiwan and provides more relevant information in conducting monetary and fiscal policies.

1. INTRODUCTION

Potential output and the non-accelerating inflation rate of unemployment (NAIRU) are defined as the level of output and unemployment rate consistent with a stable rate of inflation. The output gap is defined as the difference between actual output and potential output. When the actual output level (unemployment rate) is higher (lower) than potential output (NAIRU), excess demand drives up the inflation rate and vice versa. Thus, precise estimates of potential GDP and the NAIRU are essential for the conduct of monetary and fiscal policies.

Many methodologies have been developed in estimating potential output and the NAIRU, and a partial list includes the univariate detrending method, the multivariate filtering approach, and the structural VAR system approach.[1] To account for the stochastic trend observed for GDP, all these methods assume existence of a unit root for the unobserved potential GDP. For example, Watson (1986) decomposed observed GDP as the sum of potential GDP and the output gap, where the first component follows a random walk with drift and the second an AR(2) process. Within a structural VAR system, Apel and Jansson (1999a, 1999b) further linked output, unemployment and inflation via Okun's law and modified Phillips Curve. Potential output and the NAIRU are assumed to follow random walk with and without drift respectively. While this assumption of the unit root is appropriate for seasonally adjusted data, it could result in misleading results for seasonally unadjusted data. In Taiwan, almost all macroeconomic data are seasonally unadjusted, and seasonal characteristics have to be explicitly considered. Hence, the assumption of the unit root is of limited interest for macroeconomic policy.

This paper aims at providing appropriate measures for Taiwan's potential GDP, output gap and NAIRU. To allow for seasonal variation, a seasonal unit

[1] For examples of the univariate decomposition, see Hodrick and Prescott (1997), Beveridge and Nelson (1981) and Watson (1986). As for multivariate unobserved component, Kuttner (1994) specifies a bivariate model with output and inflation. Blanchard and Quah (1989) implement a structural VAR system in estimating potential output.

root rather than a regular unit root and richer dynamics are embedded in Watson's and Apel and Jansson's models. Empirical results confirm the importance of seasonal behavior. The regular unit root models fully transfer the seasonal variation observed in actual GDP to potential GDP and output gap, while our seasonal unit root models do not. Switching from a regular unit root to a seasonal unit root improves the efficiency of measuring potential GDP and the output gap for Taiwan. Actually, it is difficult to obtain sensible estimates for the regular unit root models.

Our results also extend the potential output literature. Previous potential output analysis views seasonality as an undesirable characteristic and relies on official seasonally adjusted data at the source. There exist two main criticisms on seasonally adjusted data. One is that discarding information about the seasonality may change dynamic correlations between macroeconomic variables (Fok et al., 2005) and distort the analysis of business cycles (see e.g., Christiano and Todd, 2002; Matas-Mir and Osborn, 2004). The other is that traditional seasonal adjustment methods, like the Census X-11 or seasonal dummies, have insufficient connection to economic theory (see e.g., Diewert, 1996).[2] We show that switching from a regular unit root to a seasonal unit root allows us to estimate potential output and the NAIRU directly with seasonally unadjusted data, and richer dynamics can be embedded in models.

The second dimension in which the basic model is extended is to impose some structure on the NAIRU and the unemployment gap. Empirical literature indicates that the estimates of the NAIRU are measured with large uncertainty (see Cross et al., 1997; Staiger et al., 1996; Richardson et al., 2000) and there is a substantial variation in the precision of NAIRU estimates, both across countries and across specifications (Laubach, 2001; Stephanides, 2006).[3] In estimating

---

[2] For example, the linearity of X-11 filter (the core of the Census X-12-ARIMA program) has been strongly criticized (see e.g., Cleveland and Tiao, 1976; Ghysels et al., 1996; Matas-Mir et al., 2008). The X-12-ARIMA, a symmetric two-sided linear moving average filter, can result in destruction of information, and this effect is irreversible. Moreover, the constant seasonality that is implicitly assumed by seasonal dummies is unrealistic and of limited interest for macroeconomic policy.

[3] For instance, Laubach (2001), Fabiani and Mestre (2001) and Denis et al. (2002) adopted models for representing the NAIRU as a random walk process with a stochastic trend. Richardson et al. (2000) even proposed an ad hoc stochastic specification that the change in the NAIRU is a first-order auto-regressive process.

Taiwan's NAIRU and unemployment gap, we find that there appears to be a rising trend in the unemployment gap during the 2000s. Also, the sum of the coefficients of the autoregressive components for the unemployment gap, $\delta(L)$, is somewhat greater than unity. To let the data speak as much as possible and to investigate how sensitive the results are to further changes in the specification of the NAIRU, distinct classes of NAIRU concept are implemented in this paper.

Further analyses show that adding a deterministic drift or stochastic drift in the NAIRU does not yield sensible estimates, and the drift is statistically insignificant. Allowing a drift in cyclical unemployment still affords no improvement. However, specifying the change in the unemployment gap as a first-order auto-regressive process leads to a dramatic decline in the sum of the coefficients of the autoregressive components of the unemployment gap and the estimated output gap, and the unemployment gap are more stable.

Unlike previous Taiwan's potential output analyses, this is the first paper that applies Apel and Jansson's model.[4] We show that incorporating simultaneously the building blocks of Okun's law and the Phillips curve indeed provides more relevant information for the regulatory authority in conducting monetary and fiscal policies. In contrast to the progressive strengthening of economic growth during 1983–1999, the period after the year 2000 was characterized by two sharp decreases in GDP. The reduction in domestic demand in 2001 pulled up the unemployment rate and pushed the output gap into negative territory. The financial crisis of 2008 subsequently caused the unemployment rate to approach 6% and further induced a sharp increase in the "equilibrium" rate of unemployment, NARIU. At the end of the sample, the NAIRU climbs to its highest level and the unemployment gap shoots up to above 1.5%.

Since the rise in the NAIRU is connected with several shocks that hit the economy, the estimated deviation of actual unemployment from the NAIRU

[4] To our knowledge, there are a few papers which attempted to estimate Taiwan's potential output, with the exceptions of Wu and Lin (2002), Gerlach and Yiu (2004) and Lin (2010). The methodologies they use include the Watson's model, the HP filter and the production function approach. However, previous literature documents that production function relies on an overly simplistic representation of the production technology approach, and the HP filter may suffer from end-of-sample bias (Cerra and Saxena, 2000). Since Watson's model is a stochastic decomposition method, it does not rely on clear theoretical foundations. Moreover, none of these papers consider the NAIRU and potential output simultaneously.

should not be explicitly attributed to structural changes. However, the Taiwan government should keep watching whether the NAIRU consistently stays at a high level or the deviation of actual unemployment from the NAIRU gets wider. These imply relatively unstable conditions in the labour market, meaning that the economy has not adjusted to a relatively high level of unemployment. When these happen, there is probably little room for countercyclical policies, but rather more room for structural policies aimed at reducing the NAIRU.

In addition to this introduction, we summarize the state space model and Kalman Filter in Section 2. Section 3 reports the empirical results and Section 4 concludes.

2. ECONOMETRIC MODELS

The state space model (SSM) has been a very powerful framework for the analysis of dynamical systems. While linear regression models use exogenous variables to distinguish the explained variation from the unexplained variation, SSM relies on the dynamics of the state variables and the linkage between the observed variables and state variables to draw statistical inference about the unobserved states. Potential GDP, the output gap and the NAIRU are all unobserved, and it is natural for us to embed our analysis in SSM. We shall start this section with a brief summary of SSM.

2.1 A Brief Summary of SSM and the Kalman Filter

The general state space model can be written as:

$$
\begin{aligned}
y_t &= c_t + Z_t\alpha_t + \varepsilon_t,\ \ \varepsilon_t \sim \mathrm{NID}\,(0, H_t)\,, \\
\alpha_{t+1} &= d_t + T_t\alpha_t + R_t\eta_t,\ \ \eta_t \sim \mathrm{NID}\,(0, Q_t)\,, \quad t = 1, \cdots, n, \\
\alpha_1 &\sim N\,(a_1, P_1)\,,
\end{aligned} \tag{1}
$$

where α_t, y_t, ε_t, η_t are the state variables, observed variables, measurement error terms, and disturbance terms respectively. c_t, d_t are exogenous variables,

Table 1 Dimensions for State Space Models

vector		matrix	
y_t	$p \times 1$	Z_t	$p \times m$
α_t	$m \times 1$	T_t	$m \times m$
ε_t	$p \times 1$	H_t	$p \times p$
η_t	$r \times 1$	R_t	$m \times r$
a_1	$m \times 1$	Q_t	$r \times r$
v_t	$p \times 1$	P_1	$m \times m$
a_t	$m \times 1$	P_t	$m \times m$

Z_t, and T_t could be time-varying or time-invariant. The dimensions of all variables are summarized in Table 1.

For initial conditions, we can either fix them at appropriate values near the final solution to speed up convergence rate or use the usual diffuse prior:

$$P = P_* + \kappa P_\infty, \tag{2}$$

where P_∞ is an $m \times m$ matrix with zeros and ones. κ is large, say $\kappa = 10^{10}$. Define

$$\begin{aligned} a_{t+1} &= \mathrm{E}\left(\alpha_{t+1} | \mathbf{Y}_t\right), \\ P_{t+1} &= \mathrm{Var}\left(\alpha_{t+1} | y_t\right), \\ v_t &= y_t - Z_t a_t, \end{aligned} \tag{3}$$

and denote $a_{t|t} = \mathrm{E}(\alpha_t | \mathbf{Y}_t)$, $P_{t|t} = \mathrm{Var}(\alpha_t | \mathbf{Y}_t)$. We have

$$\begin{aligned} a_{t|t} &= a_t + M_t F_t^{-1} v_t, \\ a_{t+1} &= T_t a_{t|t}, \\ F_t &= Z_t P_t Z_t' + H_t, \\ M_t &= P_t Z_t', \\ P_{t|t} &= P_t - M_t F_t^{-1} M_t', \\ P_{t+1} &= T_t P_{t|t} T_t' + R_t Q_t R_t', \end{aligned} \tag{4}$$

where $\mathbf{Y}_t = \sigma(y_1, y_2, \cdots, y_t)$, information up to time t. Also, denote $y = \sigma(y_1, \ldots, y_n)$, $\hat{\alpha}_t = \mathrm{E}(\alpha_t|y)$.

Then,

$$\begin{aligned}\hat{\alpha}_t &= \mathrm{E}\left(\alpha_t|y\right)\\ &= a_{t|t} + P_{t|t}T_t' P_{t+1}^{-1}\left(\hat{\alpha}_{t+1} - a_{t+1}\right)\\ &= a_t + P_t Z_t' F_t^{-1} v_t + P_t L_t' P_{t+1}^{-1}\left(\hat{\alpha}_{t+1} - a_{t+1}\right).\end{aligned} \tag{5}$$

Let

$$r_t = P_{t+1}^{-1}\left(\hat{\alpha}_{t+1} - a_{t+1}\right), \tag{6}$$

then

$$r_{t-1} = Z_t' F_t^{-1} v_t + L_t' r_t, \tag{7}$$

with $r_n = 0$. For the variance,

$$\begin{aligned}V_t &= \mathrm{Var}(\alpha_t|y)\\ &= P_t - P_t N_{t-1} P_t,\\ N_{t-1} &= Z_t' F_t^{-1} Z_t + L_t' N_t L_t,\\ r_{t-1} &= Z_t' F_t^{-1} v_t + L_t' t_t,\\ \hat{\alpha}_t &= a_t + P_t r_{t-1}.\end{aligned} \tag{8}$$

Excellent exposition of SSM can be found in Koopman et al. (1999), Durbin and Koopman (2001), Zivot et al. (2003), Tsay (2005) and Koopman and Ooms (2006).

2.2 Watson's Decomposition

Watson (1986) decomposed observed GDP as the sum of potential GDP and the output gap, and the model is listed below.

$$y_t = y_t^p + z_t, \tag{9}$$

$$y_t^p = y_{t-1}^p + \mu_y + e_{yt},\ e_{yt} \sim NID\left(0, \sigma_y^2\right), \tag{10}$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + e_{zt},\ e_{zt} \sim NID\left(0, \sigma_z^2\right), \tag{11}$$

where y_t is observed output, and y_t^p denotes potential GDP.

It is worth noting that different dynamic behavior plays the central role for the decomposition. Potential GDP follows a random walk with drift which governs the long-term behavior of y_t whereas output gap determines the short-run behavior. In order for the model to be identifiable, y_t^p cannot have short-run dynamics. For seasonally adjusted data, y_t has a unit root and so does y_t^p. Furthermore, two lags for the output gap is enough to capture the dynamics in y_t in US.

For seasonally unadjusted series, we frequently observe a seasonal unit root rather than a regular unit root. The potential GDP should be respecified accordingly. Following the line of thoughts above, we replace the random walk equation with the seasonal unit root equation and keep the specification of the output gap unchanged. The model then becomes:

$$y_t = y_t^p + z_t, \tag{12}$$

$$y_t^p = y_{t-4}^p + \mu_y + e_{yt}, \; e_{yt} \sim NID\big(0, \sigma_y^2\big), \tag{13}$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + e_{zt}, \; e_{zt} \sim NID\left(0, \sigma_z^2\right). \tag{14}$$

The corresponding SSM is:

$$y_t = [1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0] \begin{bmatrix} y_t^p \\ y_{t-1}^p \\ y_{t-2}^p \\ y_{t-3}^p \\ z_t \\ z_{t-1} \end{bmatrix}, \tag{15}$$

$$\begin{bmatrix} y_t^p \\ y_{t-1}^p \\ y_{t-2}^p \\ y_{t-3}^p \\ z_t \\ z_{t-1} \end{bmatrix} = \begin{bmatrix} \mu_y \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1}^p \\ y_{t-2}^p \\ y_{t-3}^p \\ y_{t-4}^p \\ z_{t-1} \\ z_{t-2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_{yt} \\ e_{zt} \end{bmatrix},$$

$$\delta = \begin{bmatrix} \mu_y \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Phi = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\Omega = \mathrm{Cov} \begin{bmatrix} e_{yt} \\ e_{zt} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{16}$$

As is obvious from the model above, switching from a regular unit root to a seasonal unit root introduces higher-order dynamics into the system. To cast the model in a first-order Markovian framework, more state variables (lagged values of unobserved variables) are created and inter-linked by identity equations. Some diagonal elements of the covariance matrix of the disturbance terms are consequently zeros but this imposes no problem at all for SSM.

2.3 Apel and Jansson's Decomposition

Apel and Jansson (1999a, 1999b) added inflation rates and unemployment to the model. Output and employment are linked via Okun's law while the relationship between output and inflation is governed by Phillips curve. The complete model is:

$$y_t - y_t^p = \phi(L)\left(u_t - u_t^n\right) + \epsilon_t^{ol}, \tag{17}$$
$$u_t^n = u_{t-1}^n + \epsilon_t^n, \tag{18}$$
$$y_t^p = \alpha + y_{t-1}^p + \epsilon_t^p, \tag{19}$$
$$u_t - u_t^n = \delta(L)\left(u_{t-1} - u_{t-1}^n\right) + \epsilon_t^c, \tag{20}$$

and

$$\pi_t = \rho(L)\pi_{t-1} + \eta(L)\left(u_t - u_t^n\right) + \omega(L)z_t + \epsilon_t^{pc}, \tag{21}$$

where π_t is price inflation defined as the gross rate of consumer price index, u_t is the unemployment rate measured as a percent of unemployment to labor force, y_t is the real GDP expressed as the log of real output and y_t^p denotes potential GDP. z_t, is a vector of supply-shock proxies and is measured by the contemporaneous changes in import price index in our model. The NAIRU is represented as u_t^n, and $\eta(L)$, $\omega(L)$, $\phi(L)$, $\rho(L)$ and $\delta(L)$ are polynomials in the lag operator determined by the data. All the error terms here are assumed to be independently and identically distributed.

Unlike GDP where a drift term is added, the NAIRU follows a random walk without drift. Similar to GDP, the unemployment rate also displays obvious seasonal variation, and the model of the NAIRU has to capture this seasonal pattern. Instead of a regular unit root, we assume the existence of a seasonal unit root for the NAIRU.[5]

$$y_t - y_t^p = \phi(L)\left(u_t - u_t^n\right) + \epsilon_t^{ol}, \tag{22}$$
$$u_t^n = u_{t-4}^n + \epsilon_t^n, \tag{23}$$
$$y_t^p = \alpha + y_{t-4}^p + \epsilon_t^p, \tag{24}$$
$$u_t - u_t^n = \delta(L)\left(u_{t-1} - u_{t-1}^n\right) + \epsilon_t^c, \tag{25}$$

and

$$\pi_t = u_\pi + \rho(L)\pi_{t-1} + \eta(L)\left(u_t - u_t^n\right) + \omega(L)z_t + \epsilon_t^{pc}. \tag{26}$$

[5] Note that we add a constant in equation (26) and this is a data-driven specification. In estimating inflation in Taiwan, there exists a constant in inflation and it is significant at the 5% level according to the t-statistics.

Empirical analysis finds $\phi(L) = \phi_1 + \phi_2 L$, $\eta(L) = \eta_1 + \eta_2 L$ and $\delta(L) = \delta_1 + \delta_2 L$ (see, for example, Apel and Jansson, 1999a; Basistha and Startz, 2008; Hjelm and Jonsson, 2010).[6] Thus, our model has the following vector observation equation:

$$\begin{bmatrix} y_t \\ u_t \\ \pi_t^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta_1 & \eta_2 \end{bmatrix} \begin{bmatrix} y_t^p \\ y_{t-1}^p \\ y_{t-2}^p \\ y_{t-3}^p \\ u_t^n \\ u_{t-1}^n \\ u_{t-2}^n \\ u_{t-3}^n \\ u_t - u_t^n \\ u_{t-1} - u_{t-1}^n \end{bmatrix} + \begin{bmatrix} \epsilon_t^{ol} \\ 0 \\ \epsilon_t^{pc} \end{bmatrix}, \quad (27)$$

where $\pi_t^* = \pi_t - u_\pi - \rho(L)\pi_{t-1} - \omega(L)z_t$.
The dynamics of the state variables are summarized by the following vector state equation:

$$\begin{bmatrix} y_t^p \\ y_{t-1}^p \\ y_{t-2}^p \\ y_{t-3}^p \\ u_t^n \\ u_{t-1}^n \\ u_{t-2}^n \\ u_{t-3}^n \\ u_t - u_t^n \\ u_{t-1} - u_{t-1}^n \end{bmatrix} = \begin{bmatrix} \alpha \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta_1 & \delta_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1}^p \\ y_{t-2}^p \\ y_{t-3}^p \\ y_{t-4}^p \\ u_{t-1}^n \\ u_{t-2}^n \\ u_{t-3}^n \\ u_{t-4}^n \\ u_{t-1} - u_{t-1}^n \\ u_{t-2} - u_{t-2}^n \end{bmatrix} + \begin{bmatrix} \epsilon_t^p \\ 0 \\ 0 \\ 0 \\ \epsilon_t^n \\ 0 \\ 0 \\ 0 \\ \epsilon_t^c \\ 0 \end{bmatrix}, \quad (28)$$

[6] We also conduct experiments by adding longer lags, AR(3) and AR(4). The unreported results of AR(3) are quantitatively and qualitatively similar to those reported in the context, except that the third lag of cyclical unemployment in Watson's model is not significant at the 5% level according to the t-statistics. For simplicity, we only report results of AR(2) and AR(4) in this paper.

Table 2 Estimating Results of Watson's Model

	Regular unit root		Seasonal unit root	
Parameters	IAR(2)	IAR(4)	SIAR(2)	SIAR(4)
α	0.0145 (0.0017)	0.0144 (0.0016)	0.0586 (0.0067)	0.0582 (0.0066)
ϕ_1	0.0076 (0.0210)	0.1020 (0.0734)	1.5054 (0.0222)	1.5609 (0.0243)
ϕ_2	−0.9974 (0.0212)	−0.0897 (0.0703)	−0.5101 (0.0225)	−0.6279 (0.0287)
ϕ_3	—	0.0856 (0.0744)	—	0.0393 (0.1197)
ϕ_4	—	0.9048 (0.0744)	—	0.0238 (0.1049)
σ_y	0.0179 (0.0013)	0.0163 (0.0014)	0.0050 (0.0009)	0.0051 (0.0010)
σ_z	0.0030 (0.0008)	−0.0058 (0.0013)	−0.0120 (0.0011)	0.0116 (0.0012)
Log-likelihood	276.88	279.50	289.60	289.98

Note: IAR(2) denotes the regular unit root with AR(2), IAR(4) the regular unit root with AR(4), SIAR(2) the seasonal unit root with AR(2) and SIAR(4) the seasonal unit root with AR(4).

where $\Omega \equiv Diag(\sigma_p^2, 0, 0, 0, \sigma_n^2, 0, 0, 0, \sigma_c^2, 0, \sigma_{ol}^2, 0, \sigma_{pc}^2)$.

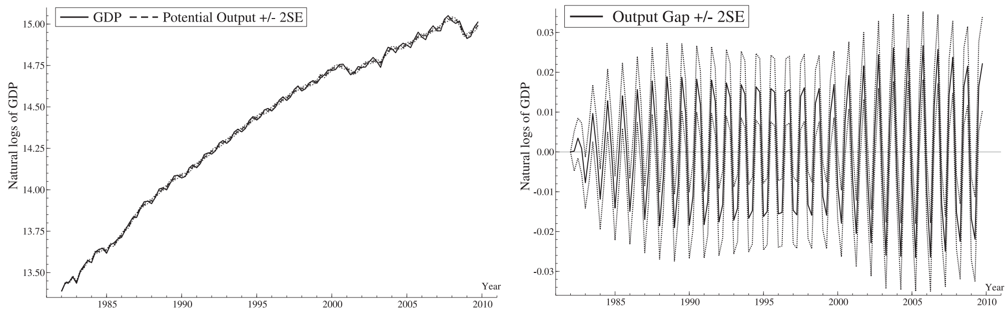
3. DATA AND EMPIRICAL RESULTS

All data are quarterly, ranging between 1982:01 to 2009:04 and mainly taken from the AREMOS database and the Taiwan Economic Journal (TEJ). The models are estimated using *SsfPack* of OX, written by Siem Jan Koopman, Neil Shephard and Jurgen A. Doornik. See Koopman et al. (1999).

3.1 Results for Watson's Model

We first investigate the impact of seasonality on Watson's model. Table 2 indicates the results for the four specifications of Watson's model. The second column reports the results of Watson's regular model with an AR(2) output gap and the figure is put in Panel A of Figure 1. As is obvious from the figure, the output gap fluctuates around 0 rapidly, with a strong seasonal pattern. To check if adding longer lags to the regular unit root model could cure the problem, we re-estimate Watson's model with an AR(4) output gap. The estimation

Panel A: Watson's model with a regular unit root and an AR(2) output gap



Panel B: Watson's model with a regular unit root and an AR(4) output gap

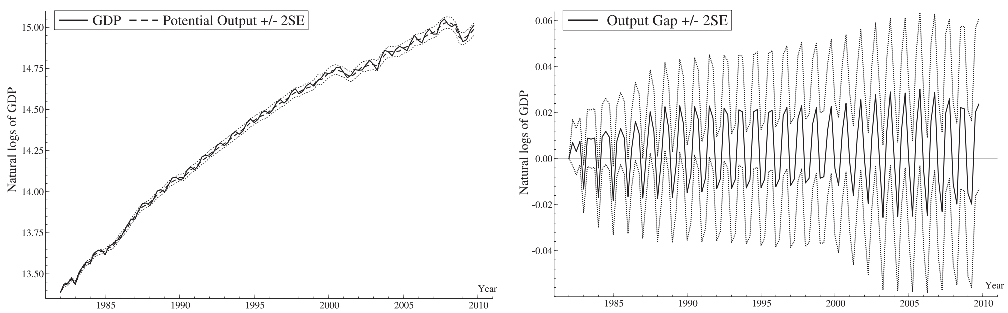
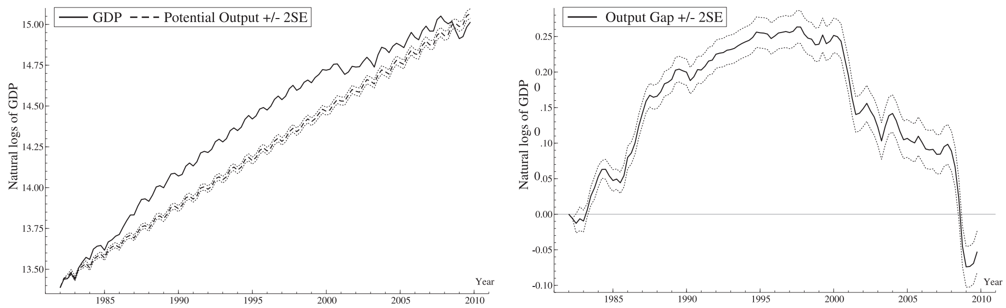


Figure 1 Watson's Model with a Regular Unit Root

results are put in the third column of this table and Panel B of Figure 1. Clearly, it does not work.

We then proceed with estimating Watson's model with a seasonal unit root. Considering the parameter uncertainty, we calculate 95% confidence bounds around the potential output. The result is put in the fourth and fifth columns of the Table 2 and Figure 2. From the figure, the output gap behaves nicely as expected. There exists no more seasonal variation. The output gap is negative in the early 1980s and the late 2000s, and positive between the mid 1980s and the early 2000s. The Taiwan economy experienced two sharp decreases in GDP during the 2000s. The narrower confidence bounds indicate that recessions are well identified. The similarity between the seasonal unit root with AR(2) and AR(4) for short-run dynamics and the insignificant values of ϕ_3 and ϕ_4 in Table 2 suggest that our seasonal unit root model fits the data well, with low order.

Panel A: Watson's model with a seasonal unit root and an AR(2) output gap



Panel B: Watson's model with a seasonal unit root and an AR(4) output gap

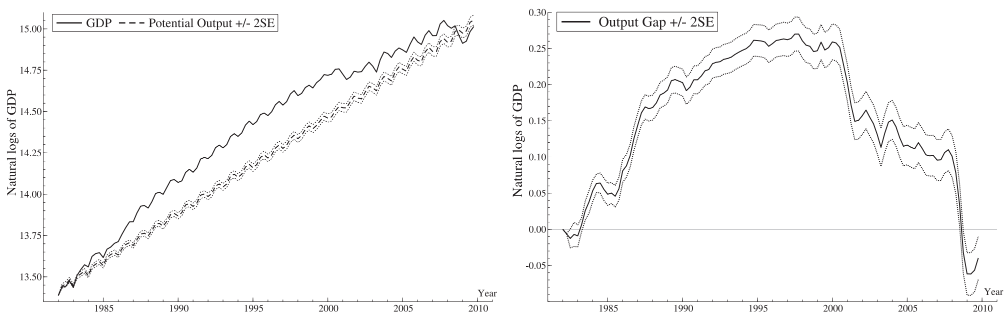


Figure 2 Watson's Model with a Seasonal Unit Root

It is worth mentioning that the final estimates for regular unit root models are somewhat sensitive to the initial condition but much more stable for seasonal unit root models.

We now reexamine our results from an economic perspective. During 1983–1999, Taiwan experienced a progressive strengthening of economic growth and sustained a growth rate in GDP of 7 to 12 percent. By contrast, the period after year 2000 was characterized by two sharp decreases in GDP. The first decrease in GDP in 2001 was associated with a relatively low growth rate in private consumption and gross investment. Especially for gross investment, the yearly growth rate of gross investment was −26.83%, and it slowed down for subsequent years. Thus, Taiwan encountered its first negative GDP growth rate in 2001. Furthermore, the financial crisis of 2008 resulted in a more severe recession. Most rich economies were facing recession, and this contributed to an export-led economic recession in Taiwan. Since September 2008, Taiwan has experienced a persistently fourteen-month negative growth rate in exports. Toward the end of the sample, the contemporaneous monthly growth rate of exports is −30.38%. The gradual decline in private consumption and investment, combined with the abrupt reduction in the economy's exports, finally caused the output gap to become negative.

3.2 Results for Apel and Jansson's Model

Now we turn to extend the seasonal unit root to Apel and Jansson's model. Results from estimating each of the four specifications for Apel and Jansson's model are reported in columns 1 to 4 of Table 3, with IAR(4) denoting the regular unit root with AR(4), SIAR(2) the seasonal unit root with AR(2), SIAR(4) the seasonal unit root with AR(4), and SIDAR(1) the seasonal unit root with differenced unemployment gap and AR(1). The corresponding estimates and 95% confidence intervals are plotted in Figures 3 to 6.[7]

As expected, Apel and Jansson's model with AR(2) specification for output and the unemployment gap does not yield sensible estimates. Actually, convergence is seldom achieved and the final estimates are sensitive to the initial values. Thus, we turn to the model with AR(4) specification and report the estimation results in the second column of Table 3 and Figure 3. Obviously, the unit root specification transfers the strong seasonality from the observed data to the output gap and the NAIRU. This is certainly not a desired feature since these two terms are defined over long-term behavior and should exclude seasonality.

However, when the seasonal unit root is explicitly considered, the estimates of the model no longer exhibit seasonal behavior and the parameter estimates are generally in accordance with economic theory. The estimation results for a seasonal unit root with AR(2) and AR(4) are put in columns 3 and 4 of Table 3 and Figures 4 and 5, respectively. Table 3 indicates that the sum of the coefficients on cyclical unemployment is negative in both the Phillips curve and in the Okun's law relationship, and most individual parameter estimates are

[7] For simplicity, some specifications that do not yield sensible estimates or achieve convergence are not reported in this paper.

Table 3 Estimation Results for Apel and Jansson's Model

	Regular Unit Root		Seasonal Unit Root	
Parameter	IAR(4)	SIAR(2)	SIAR(4)	SIDAR(1)
α	0.0165 (0.0013)	0.0754 (0.0091)	0.0710 (0.0039)	0.0628 (0.0028)
δ_1	0.9040 (0.0016)	1.9343 (0.0133)	2.7309 (0.0041)	0.8620 (0.0007)
δ_2	−0.4228 (0.0012)	−0.9328 (0.0145)	−2.5251 (0.0041)	—
δ_3	0.6683 (0.0012)	—	0.7648 (0.0046)	—
δ_4	0.0923 (0.0008)	—	0.0338 (0.0055)	—
ϕ_1	−0.0657 (0.0011)	2.1122 (0.0156)	−1.0726 (0.0070)	−0.5023 (0.0009)
ϕ_2	−0.1104 (0.0010)	−2.8399 (0.0267)	0.2528 (0.0038)	0.3397 (0.0007)
ϕ_3	−0.0061 (0.0011)	—	2.7345 (0.0040)	—
ϕ_4	0.0034 (0.0014)	—	−2.2450 (0.0048)	—
η_1	1.9870 (0.0013)	2.2076 (0.0195)	−0.7661 (0.0040)	−4.5666 (0.0006)
η_2	−0.0653 (0.0011)	−3.0534 (0.0140)	1.1677 (0.0041)	3.9371 (0.0006)
η_3	−1.1158 (0.0011)	—	−2.2303 (0.0053)	—
η_4	0.6877 (0.0013)	—	1.2336 (0.0067)	—
u_π	0.4404 (0.1423)	0.4404 (0.1423)	0.4404 (0.1423)	0.4404 (0.1423)
ρ_1	0.7797 (0.0950)	0.7797 (0.0950)	0.7797 (0.0950)	0.7797 (0.0950)
ρ_2	0.0086 (0.1206)	0.0086 (0.1206)	0.0086 (0.1206)	0.0086 (0.1206)
ρ_3	0.1243 (0.1198)	0.1243 (0.1198)	0.1243 (0.1198)	0.1243 (0.1198)
ρ_4	−0.1785 (0.0776)	−0.1785 (0.0776)	−0.1785 (0.0776)	−0.1785 (0.0776)
ω_1	0.0187 (0.0140)	0.0187 (0.0140)	0.0187 (0.0140)	0.0187 (0.0140)
σ_p	0.0126 (0.0012)	0.0043 (0.0090)	0.0044 (0.0009)	0.0041 (0.0005)
σ_n	13.8380 (0.0011)	0.6137 (0.0499)	0.5918 (0.0047)	0.4863 (0.0006)
σ_c	0.1100 (0.0013)	0.0051 (0.0004)	0.0057 (0.0005)	0.0280 (0.0010)
σ_{ol}	0.0059 (0.0012)	2.24e-7 (0.0020)	1.15e-5 (0.0028)	0.0002 (0.0007)
σ_{pc}	10.6800 (0.0010)	0.9873 (0.0182)	1.0308 (0.0044)	1.0042 (0.0008)
Log likelihood	−509.56	20.02	17.00	40.97

Noet: IAR(4) denotes the regular unit root with AR(4), SIAR(2) the seasonal unit root with AR(2), SIAR(4) the seasonal unit root with AR(4) and SIDAR(1) the seasonal unit root with differenced unemployment gap and AR(1).

statistically significant. The narrower confidence bounds indicate that SIAR(4) specification produces more precise estimates for potential GDP, the NAIRU and the unemployment gap. Moreover, the maximized value of the log likelihood improves by approximately 15 percent when considering two additional lags. These reflect our beliefs that a SIAR(4) specification is more reliable than

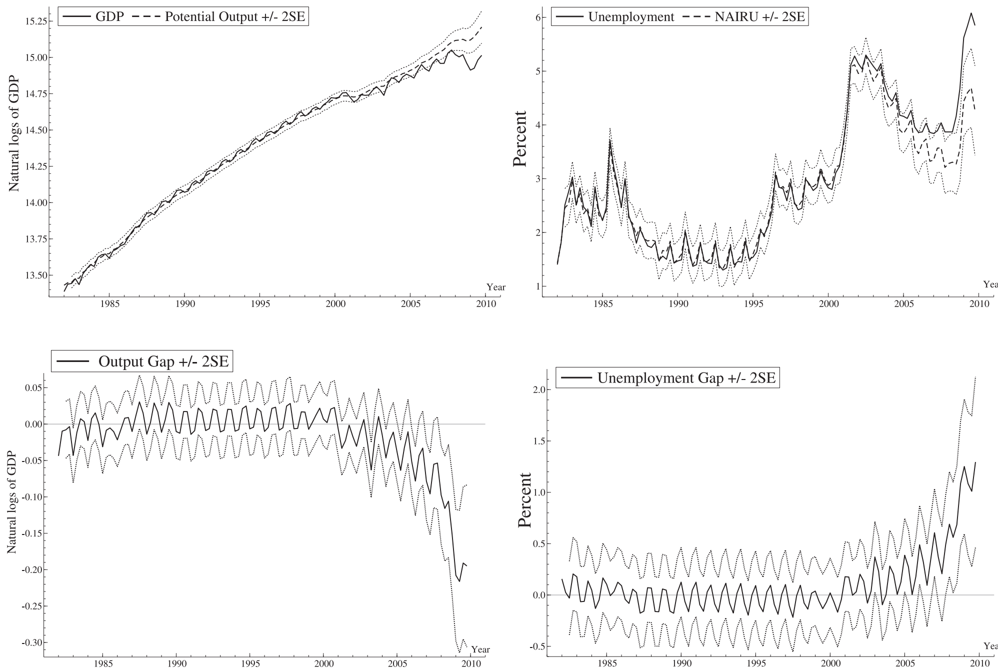


Figure 3 Apel and Jansson's Model with Regular Unit Root and AR(4)

a SIAR(2) specification. Thus, we will focus our attention on SIAR(4) specification.

From 1987 to 1997 actual unemployment was below the NAIRU. The decrease in the NAIRU implies a proportional rise in potential output growth and thus a positive output gap. According to Wu and Lin (2002), the decrease in the NAIRU during 1987 to 1996 was associated with persistent growth in manufacturing and service industries. Thus, inflationary pressures arose from the labour market and pushed up inflation at a level around 3 to 5 percent for those years.

Although the evolution of potential output obtained from Apel's model and Watson's model share some common features, there are also substantial differences. One is the different dynamic of the output gap at the beginning of the 1980s. The other is that the recession in the 2000s appears very severe compared to Watson's univariate model. In both cases, the differences can be

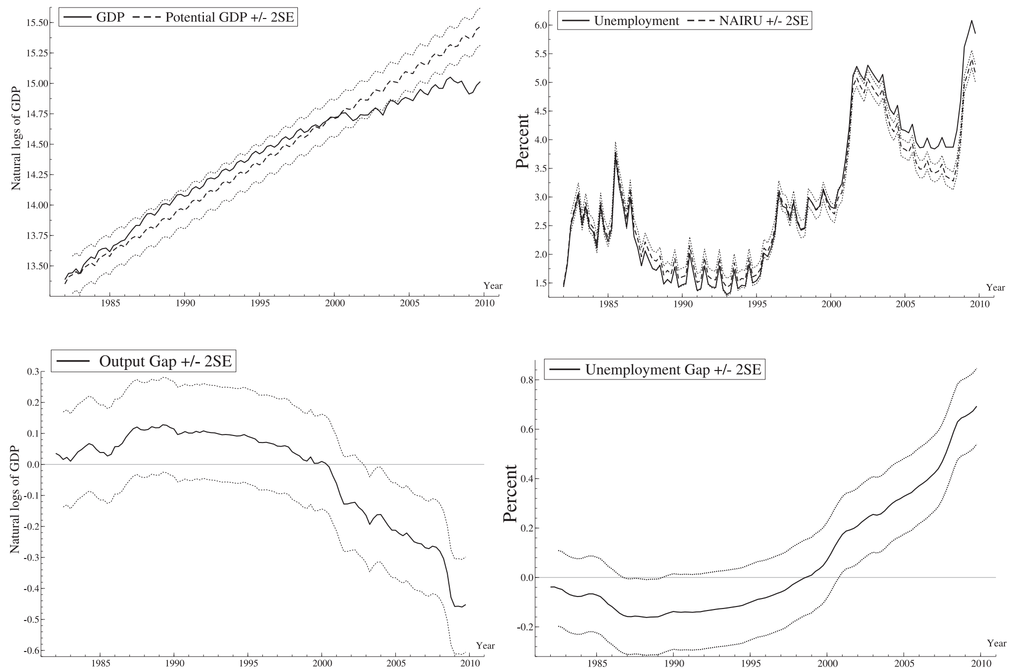


Figure 4 Apel and Jansson's Model with a Seasonal Unit Root and AR(2)

explained by the positive unemployment gap during these episodes. For example, the reduction in domestic demand in 2001 pulled up the unemployment rate and pushed the output gap into negative territory. The financial crisis of 2008 subsequently caused the unemployment rate to approach 6% and further induced a sharp increase in the equilibrium rate of unemployment, NARIU. At the end of the sample, the NAIRU climbs to its highest level and the unemployment gap shoots up to above 1.5%.

One may question whether a purely autoregressive process accurately describes the deviation of the observed unemployment rate from the NAIRU. Looking at Figure 4 or 5, there appears to be a rising trend in the unemployment gap in the 2000s. Table 3 indicates that the sum of the coefficients of the autoregressive components of the unemployment gap, $\delta(L)$, is greater than unity as well. To mitigate this concern and investigate how sensitive the results are to further changes in the specification of the NAIRU, we impose some structure on the NAIRU and the unemployment gap.

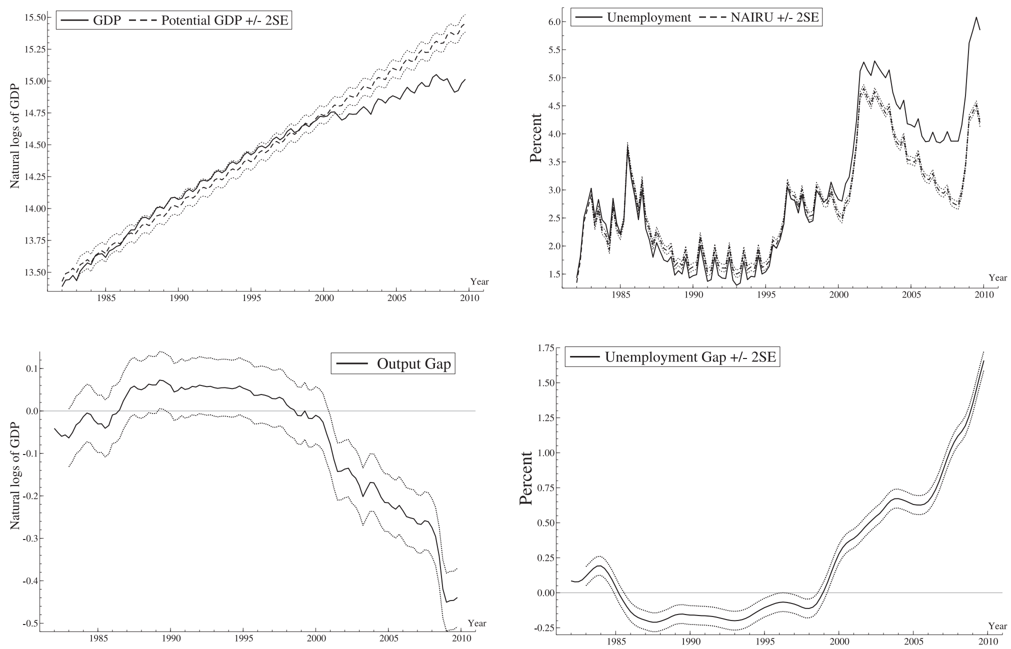


Figure 5 Apel and Jansson's Model with a Seasonal Unit Root and AR(4)

Following previous literature, we adopt equation (23) by adding a deterministic drift and a stochastic drift, respectively (see Laubach, 2001; Fabiani and Mestre, 2001; Denis et al., 2002). The unreported results indicate that adding a drift or a stochastic trend does not yield sensible estimates, and the drift is statistically insignificant. Motivated by a desire to let the data speak as much as possible, the alternative is to apply some ad hoc stochastic specifications of the unemployment gap. Since allowing a drift in cyclical unemployment, equation (25), still affords no improvement, we proceed to specify the change in unemployment gap as a first-order autoregressive process.[8] This approach followed here is similar to Richardson et al. (2000), and the estimation results are put in the last column of Table 3 and Figure 6.[9]

[8] This means that equation (25) is replaced by:

$$\Delta U_t^G = \delta(L)\Delta U_{t-1}^G + \epsilon_t^c, \text{ where } U_t^G = u_t - u_t^n. \tag{29}$$

[9] Here we only report results of the seasonal unit root with the differenced unemployment

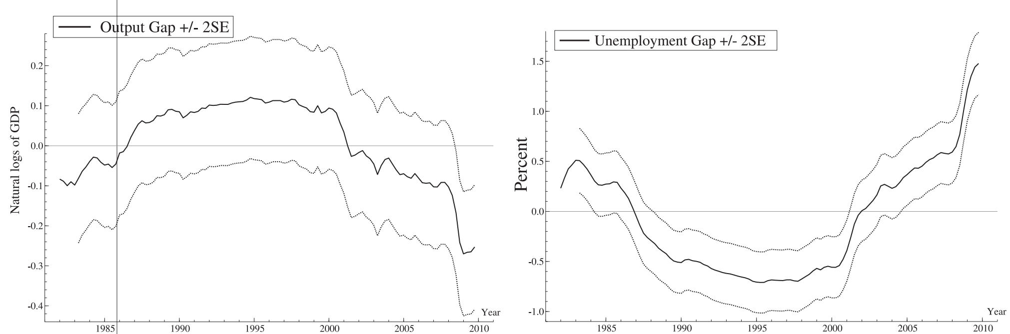


Figure 6 Apel's Model with a Seasonal Unit Root and a Differenced Unemployment Gap with AR(1)

From Table 3 and Figure 6, we make the following observations. First, specifying the change in the unemployment gap as a first-order auto-regressive process leads to a dramatic decline in the sum of the coefficients of the autoregressive components of the unemployment gap. The sum of δ's is now much less than unity which ensures that the change in unemployment gap has a tendency to revert to its mean over time. Moreover, the final estimates for a seasonal unit root with the differenced unemployment gap are less sensitive to the initial condition and much more stable among four specifications. Second, our seasonal unit root model is robust to the changes in the specification of cyclical unemployment. Detrending the unemployment gap gives a very similar result in comparison to the basic model. Indeed, as seen in Figure 6, a hump-shaped

---

gap and AR(1). We also conduct experiments by adding longer lags, AR(2). The unreported result is quantitatively and qualitatively similar to AR(1). However, the second lag is not significant at the 5% level according to the t-statistics. This implies that including one lag of differenced unemployment gap is sufficient.

output gap is observed and there exists an asymmetric (negative) correlation between the unemployment gap and the output gap. Compared to Figure 5, the output gap and the unemployment gap displayed in Figure 6 are more stable, especially for the output gap,which fluctuates in a narrow range around -0.3 to 0.2. This result appears reasonable since the trend component has already been removed by difference. Finally, the wider confidence bounds indicate that the alternative definition of the differenced output gap produces more uncertainty in estimating potential output and output gap.

It should be added that the estimated deviation of actual unemployment from the NAIRU should not be explicitly attributed to structural changes such as worsening labour-market performance and decreasing efficiency. Since the critical period of the rise in the NAIRU is connected with several shocks that hit the economy, the periodicity of the cycle may become longer. Thus, the short-run dynamics show some appearance of an evolving trend. Although the rise in the NAIRU and unemployment rate seems to slow down in the last quarter of 2009, the unemployment gap still remains at high levels and approaches 1.75%. To determine whether a sharp increase in the unemployment rate is caused by structural changes or by the changing business cycle position, we may need to consider a longer sample period. This opens room for future research. However, the Taiwan government should keep watching whether the NAIRU consistently stays at a high level or the deviation of actual unemployment from the NAIRU gets wider. These imply relatively unstable conditions in the labour market, meaning that the economy has not adjusted to a relatively high level of unemployment. When these happen, there is probably little room for countercyclical policies, but rather more room for structural policies aimed at reducing the NAIRU.

4. CONCLUSIONS

Seasonality is an important phenomenon for seasonally unadjusted data and has to be explicitly considered when estimating the potential GDP, the output gap and the NAIRU. By introducing lagged variables, we convert the low-dimension high-order models to high-dimension low-order models so that the conventional Kalman Filtering is applicable. We compare estimates from the regular unit root model with the seasonal unit root model using Taiwan's data, as almost all of its macroeconomic data are seasonally unadjusted.

Our empirical analyses confirm the conjecture that without proper treatment of seasonal factors, no sensible estimates can be obtained. Switching from a regular unit root to a seasonal unit root indeed improves the efficiency of measuring potential GDP and the output gap for Taiwan. Since there appears to be a rising trend in the unemployment gap during 2000s, distinct classes of the NAIRU concept are implemented in this paper. Further analyses indicate that our seasonal unit root model is robust to the changes in the specification of cyclical unemployment. Detrending the unemployment gap gives a very similar result in comparison to the basic model.

This study also provides relevant information for the regulatory authority in conducting monetary and fiscal policies. We find that the period after the year 2000 was characterized by two sharp decreases in GDP and a consistent rising in unemployment gap. Compared to the NAIRU in the early 1980s, the end-of-sample estimates of the NAIRU have risen on average by almost 3 percent. No matter whether this sharp increase in the unemployment rate is caused by structural changes or by the changing business cycle position, the Taiwan government should keep watching whether the NAIRU consistently stays high without reverse. When these happen, there is probably little room for countercyclical policies, but rather more room for structural policies aimed at reducing the NAIRU. Of course, this opens room for future research.

REFERENCES

Apel, M. and P. Jansson (1999a), "System Estimates of Potential Output and NAIRU," *Empirical Economics*, 24(3), 373–388.

Apel, M. and P. Jansson (1999b), "A Theory-Consistent Approach for Estimating Potential Output and the NAIRU," *Economic Letters*, 64(3), 271–275.

Basistha, A. and R. Startz (2008), "Measuring the NAIRU with Reduced Uncertainty: A Multiple Indicator-Common Cycle Approach," *The Review of Economics and Statistics*, 90(4), 805–811.

Beveridge, S. and C. R. Nelson (1981), "A New Approach to Decomposition of Economic Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle," *Journal of Monetary Economics*, 7(2), 151–174.

Blanchard, O. J. and D. Quah (1989), "The Dynamic Effects of Aggregate Demand and Aggregate Supply," *The American Economic Review*, 79(4), 655–673.

Cerra, V. and S. C. Saxena (2000), "Alternative Methods of Estimating Potential Output and the Output Gap: An Application to Sweden," *IMF Working Paper*, No. 59.

Christiano, L. J. and R. M. Todd (2002), "The Conventional Treatment of Seasonality in Business Cycle Analysis: Does It Create Distortions?" *Journal of Monetary Economics*, 49(2), 335–364.

Cleveland, W. P. and G. C. Tiao (1976), "Decomposition of Seasonal Time Series: A Model for the Census X-11 Program," *Journal of the American Statistical Association*, 71(355), 581–587.

Cross, R., J. Darby, and J. Ireland (1997), "Uncertainties Surrounding Natural Rate Estimates in the G7," Working Paper, University of Glasgow.

Denis, C., K. Mcmorrow, and W. Roger (2002), "Production Function Approach to Calculating Potential Growth and Output Gaps Estimates for the EU Member States and the US," Economic Papers, European Commission.

Diewert, W. E. (1996), "Seasonal Commodities, High Inflation and Index Number Theory," Discussion Paper, University of British Columbia.

Durbin, J. and S. J. Koopman (2001), *Time Series Analysis by State Space Methods*, Oxford: Oxford University Press.

Fabiani, S. and R. Mestre (2001), "A System Approach for Measuring the Euro Area NAIRU," *ECB Working Paper*, No. 65.

Fok, D., P. H. Franses, and R. Paap (2005), "Performance of Seasonal Adjustment Procedures: Simulation and Empirical Results," Econometric Institute Report, Erasmus University Rotterdam.

Gerlach, S. and M. S. Yiu (2004), "Estimating Output Gaps in Asia: A Cross-Country Study," *Journal of Monetary Economics*, 18(1), 115–136.

Ghysels, E., C. W. Granger, and P. Siklos (1996), "Is Seasonal Adjustment a Linear or Nonlinear Data Filtering Process?" *Journal of Business and Economic Statistics*, 14(3), 374–386.

Hjelm, G. and K. Jonsson (2010), "In Search of a Method for Measuring the Output Gap of the Swedish Economy", Working Paper, National Institute of Economic

Research.

Hodrick, R. J. and E. C. Prescott (1997), "Postwar U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit and Banking*, 29(1), 1–16.

Koopman, S. J. and M. Ooms (2006), "Forecasting Daily Time Series Using Periodic Unobserved Components Time Series Models," *Computational Statistics and Data Analysis*, 51(2), 885–903.

Koopman, S. J., N. Shephard, and J. A. Doornik (1999), "Statistical Algorithms for Models in State Space Using SsfPack 2.2," *Econometrics Journal*, 2(1), 107–160.

Kuttner, K. N. (1994), "Estimating Potential Output as a Latent Variable," *Journal of Business and Economic Statistics*, 12(3), 361–368.

Laubach, T. (2001), "Measuring the NAIRU: Evidence from Seven Economies," *Review of Economics and Statistics*, 83(2), 218–231.

Lin, T. F. (2010), "The Effect of Global Financial Crisis on Taiwanese Potential Output," Working Paper, Council for Economic Planning and Development.

Matas-Mir, A. and D. R. Osborn (2004), "Seasonal Adjustment and the Detection of Business Cycle Phases," *ECB Working Paper*, No. 357.

Matas-Mir, A., D. R. Osborn, and M. J. Lombardi (2008), "The Effect of Seasonal Adjustment on the Properties of Business Cycle Regimes," *Journal of Applied Econometrics*, 23(2), 257–278.

Richardson, P., L. Boone, C. Giorno, M. Meacci, D. Rae, and D. Turner (2000), "The Concept, Policy Use and Measurement of Structural Unemployment: Estimating a Time Varying NAIRU across 21 OECD Countries", *OECD Working Papers*, No. 250.

Staiger, D., J. H. Stock, and M. W. Watson (1996), "How Precise are Estimates of the Natural Rate of Unemployment?" *NBER Working Paper*, No. 5477.

Stephanides, G. (2006),"Measuring the NAIRU: Evidence from the European Union, USA and Japan," *International Research Journal of Finance and Economics*, 1(3), 29–35.

Tsay, R. S. (2005), *Analysis of Financial Time Series*, Hoboken, NJ: Wiley-Interscience.

Watson, M. W. (1986), "Univariate Detrending Methods with Stochastic Trends," *Journal of Monetary Economics*, 18(1), 49–75.

Wu, C. S. and J. L. Lin (2002), "Estimating Potential GDP for Taiwan and Its Application to Policies," *Industry of Free China*, 92(10), 1–34. (in Chinese)

Zivot, E., J. Wang, and S. J. Koopman (2003), "State Space Modeling in Macroeconomics and Finance Using SsfPack for S+FinMetrics," Working Paper, University of Washington.

推估臺灣未經季節調整之潛在產出

林金龍*
國立東華大學財務金融學系

陳馨蕙
國立東華大學經濟學系
暨中華經濟研究院經濟展望中心

關鍵詞: 潛在產出、產出缺口、非加速通貨膨脹失業率、季節單根、歐肯法則、菲力浦曲線
JEL 分類代號: E23, E24, E62

* 聯繫作者: 林金龍, 國立東華大學財務金融學系, 花蓮縣 974 壽豐鄉志學村大學路二段 1 號。電話: (03) 863-3146; 傳眞: (03) 863-3130; E-mail: jlin@mail.ndhu.edu.tw。本文承國科會補助 (計畫編號: NSC-95-2415-H-001-021), 謹誌謝忱。另感謝兩位匿名評審提供諸多寶貴意見, 文中如有疏漏謬誤, 當屬作者之責。

摘　要

有效評估潛在產出與產出缺口一直是執行貨幣與財政政策的重要環節。過去文獻中用以估計潛在產出的方法大多假設在潛在產出與非加速通貨膨脹失業率 (NAIRU) 中存在一般單根 (regular unit root)。然而, 這樣的假並不適用於估計臺灣未經季節調整的潛在產出。有鑑於此, 本文以 Watson's (1986) decomposition 與 Apel and Jansson's (1999a) system approach 爲例, 以季節單根取代一般單根, 從而考量更多的季節波動於模型中。同時, 爲了驗證模型的可靠性 (robustness) 以及更爲有效評估臺灣的潛在產出與 NAIRU, 我們亦嘗試對 NAIRU 與失業缺口 (unemployment gap) 作不同的模型設定。實證結果顯示在一般單根假設下, 亦即模型未考量季節波動時, 資料本身的季節波動嚴重干擾估計結果且無法提供有用的訊息; 反之, 當以季節單根取代一般單根後確實更能有效評估臺灣潛在產出與產出缺口, 並提供更爲攸關的訊息供臺灣政府執行政策之參考。